Shear and Bending Moment Diagrams

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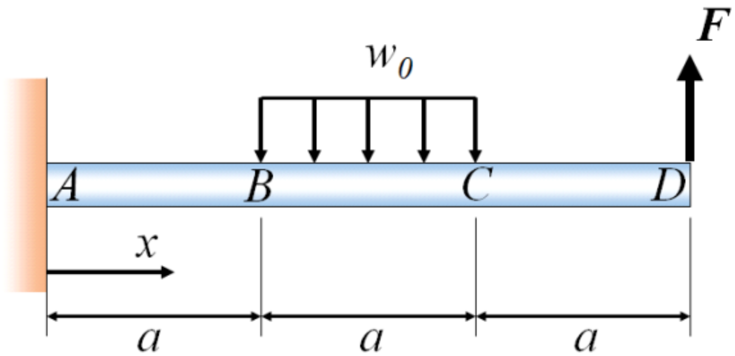
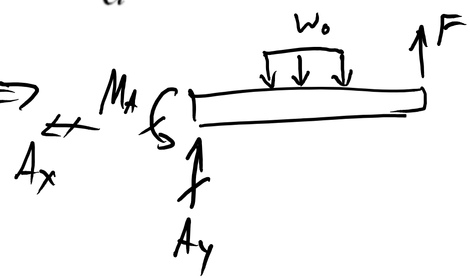
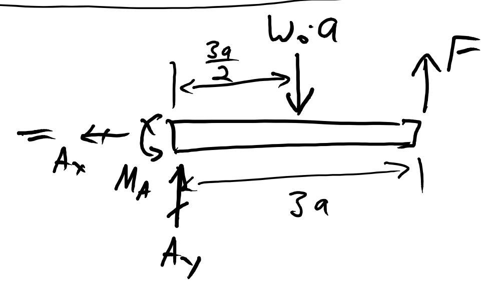


Find $V(x)$ & $M(x)$ in $a < x < 2a$

Cantilever $\Rightarrow$



$\Rightarrow$



$$\Sigma F_y = 0 \Rightarrow A_y - W_0 \cdot a + F = 0 \Rightarrow \boxed{A_y = W_0 \cdot a - F}$$

$$(\Sigma M)_a = 0 \Rightarrow M_A - \left(\frac{3a}{2}\right) W_0 \cdot a + 3 \cdot a \cdot F = 0$$

$$\boxed{M_A = \frac{3}{2} W_0 \cdot a^2 - 3 \cdot a \cdot F}$$

Cuts and FBDs

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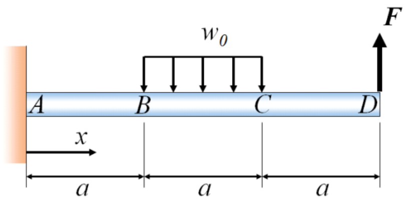
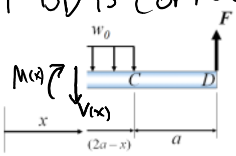
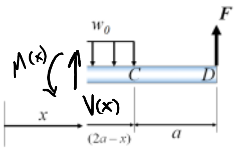
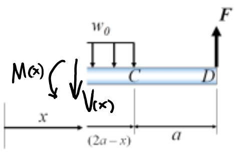
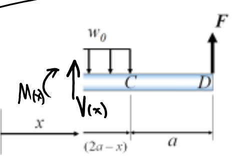


Which FBD is correct?

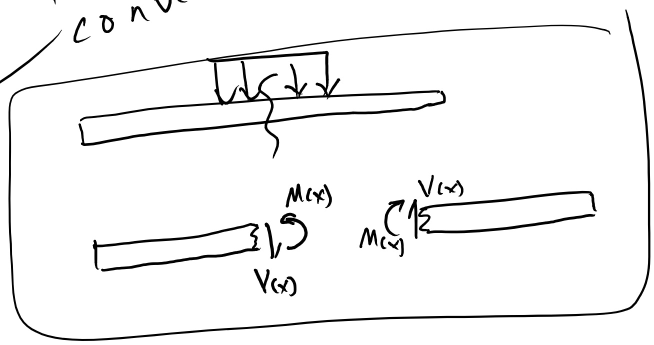
A) 

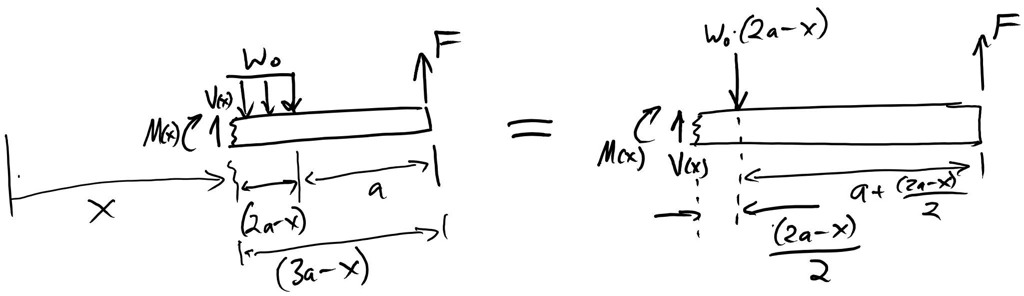
B) 

C) 

D) 

follow the sign convention





$$\Sigma F_y = 0 \Rightarrow V(x) - W_0\cdot(2a-x) + F = 0$$

$$V(x) = W_0\cdot(2a-x) - F$$

$$\boxed{V(x) = -W_0\cdot x + (2a\cdot W_0 - F)}$$

$$(\Sigma M)_x = 0 \Rightarrow -M(x) - W_0(2a-x)\frac{(2a-x)}{2} + F\cdot(3a-x) = 0$$

solve for $M(x)$

$$\boxed{M(x) = -\frac{W_0}{2}x^2 + (2a\cdot W_0 - F)x + (3aF - 2W_0\cdot a^2)}$$

$a < x < 2a$

dM

$\overline{dx} = V(x)$

Hydrostatic Forces and Buoyancy

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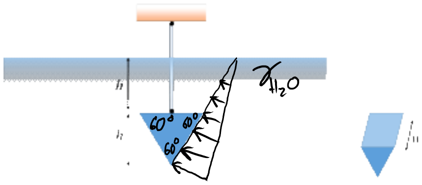
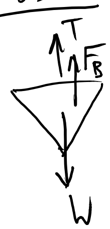


Triangular block has specific weight $\gamma = 2\gamma_{H_2O}$

Find the force in the link that supports the block

FBD



Which direction does ____ act?

A) up

B) down

Weight?

Tension in the link?

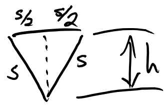
Buoyancy?

$$\Sigma F_y = 0 \Rightarrow T + F_B - W = 0$$

$$T = W - F_B$$

Find W, find F_B

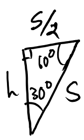
$$W = \gamma_{block} \cdot V_{block}$$



$$V_{block} = A_{triangle} \times w$$

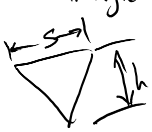
$$= \frac{1}{2} s \cdot h \cdot w$$

$$= \frac{h^2 w}{\sqrt{3}}$$



$$\Rightarrow s = \frac{2h}{\sqrt{3}}$$

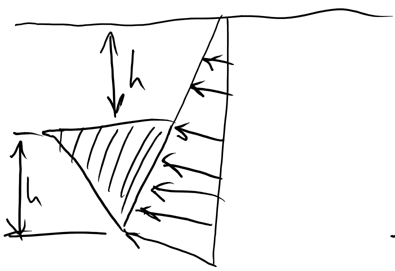
$$A_{triangle} = \frac{1}{2} \text{base} \times \text{height}$$



$$W = \gamma_{block} \cdot \frac{h^2 w}{\sqrt{3}} = 2 \cdot \gamma_{H_2O} \cdot \frac{h^2 w}{\sqrt{3}}$$

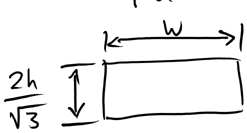
$$F_B = \gamma_{H_2O} \cdot V_{block} = \gamma_{H_2O} \frac{h^2 w}{\sqrt{3}}$$

$$\boxed{T = W - F_B = \gamma_{H_2O} \cdot \frac{h^2 w}{\sqrt{3}}}$$



Find the force F_R that acts on the inclined side, & the location it acts.

Face is rectangular



$$I = \frac{\text{base} \times \text{height}^3}{12} = \frac{w\left(\frac{2h}{\sqrt{3}}\right)^3}{12} = \frac{2wh^3}{9\sqrt{3}}$$

Resultant force: $F_R = (p_o + \gamma \cdot h_c) \cdot A$

$p_o = 0$ — H_2O — depth at the — area of the surface

Resultant force: $F_R = (p_0 + \gamma h_c) A$

$p_0 = 0$ pressure at surface

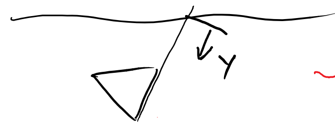
H_2O

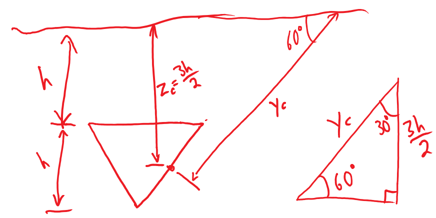
depth at the geometric centroid of A

area of the surface

$$F_R = \left[0 + \gamma_{H_2O} \cdot \left(h + \frac{h}{2}\right)\right]\left(\frac{2hw}{\sqrt{3}}\right)$$

$$\boxed{F_R = \sqrt{3} \cdot \gamma \cdot h^2 \cdot w}$$





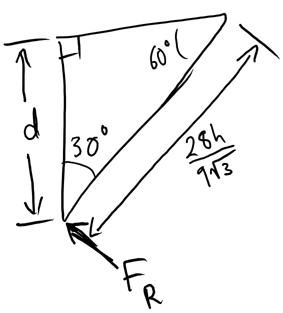
F_R acts at $y_p = y_c + \dfrac{I}{y_c \cdot A}$

$$\Rightarrow \frac{3h}{2} = y_c \cdot \cos(30°) = y_c \cdot \frac{\sqrt{3}}{2} \Rightarrow y_c = \sqrt{3} \cdot h$$

y_c is location on y-axis of geometric centroid

$$\boxed{y_p = \sqrt{3}h + \frac{\left(\frac{2\cancel{w}h^{\cancel{3}}}{9\sqrt{3}}\right)}{\cancel{\sqrt{3}} \cdot \cancel{h}\left(\frac{2\cancel{h}\cancel{w}}{\cancel{\sqrt{3}}}\right)}} = \sqrt{3} \cdot h + \frac{h}{9\sqrt{3}}$$

$$= \boxed{\frac{28h}{9\sqrt{3}}}$$



$$d = \frac{28h}{9\sqrt{3}} \cos(30°)$$
$$= \frac{28h}{9\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = \frac{14 \cdot h}{9}$$

2nd Moment of Area and Parallel Axis Theorem

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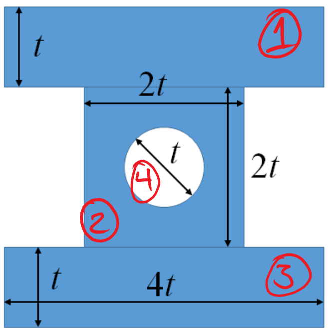
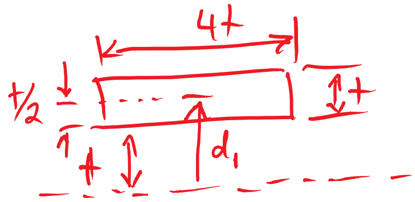


Find I of the area

$$I = I_1 + I_2 + I_3 - I_4$$

I_1



d_1 is distance from centroid of 1 to the centroid of the entire area

$$d_1 = t + \frac{t}{2} = 1.5t$$

$$A_1 = 4t^2$$

$$I_1 = \frac{(4t)t^3}{12} + d_1^2 \cdot A_1$$

$$= \frac{t^4}{3} + (t + t/2)^2 (4t^2)$$

$$= \frac{t^4}{3} + (1.5t)^2(4t^2) = \frac{28}{3}t^4$$

$$I_2 = \frac{(2t)(2t)^3}{12} = \frac{4}{3}t^4$$

$$I_3 = I_1 = \frac{28}{3}t^4$$

$$I_4 = \frac{\pi}{4}r^4 = \frac{\pi}{4}(t/2)^4 = \frac{\pi t^4}{64}$$

$$I = t^4 \cdot \left[\frac{28}{3} + \frac{4}{3} + \frac{28}{3} - \frac{\pi}{64}\right]$$

$$\boxed{I = t^4 \cdot \left(20 - \frac{\pi}{64}\right)}$$

3D Equilibrium

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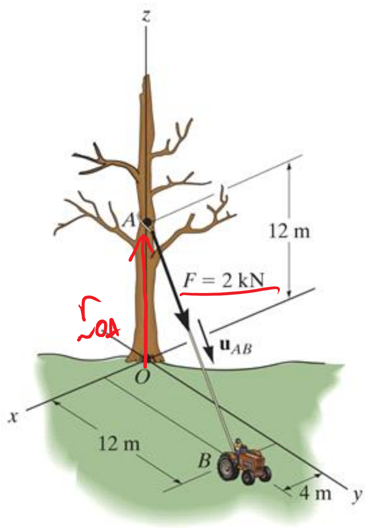

Find moment about O

$$\underset{\sim}{F} = F \cdot \hat{u}_{AB}$$

magnitude ↑ ↑ unit vector

$$\hat{u}_{AB} = \frac{\underset{\sim}{r}_{AB}}{|\underset{\sim}{r}_{AB}|}$$

$$\underset{\sim}{r}_{AB} = (4\hat{i} + 12\hat{j} - 12\hat{k})\,m$$

$$|\underset{\sim}{r}_{AB}| = \sqrt{4^2 + 12^2 + (-12)^2}\ m$$

$$= \sqrt{304}\ m$$

$$\hat{u}_{AB} = \frac{4\hat{i} + 12\hat{j} - 12\hat{k}}{\sqrt{304}}$$

$$\Rightarrow \underset{\sim}{F} = (2kN)\left(\frac{4\hat{i} + 12\hat{j} - 12\hat{k}}{\sqrt{304}}\right)$$

$$\underset{\sim}{M}_O = \underset{\sim}{r}_{OA} \times \underset{\sim}{F} \quad ; \quad \underset{\sim}{r}_{OA} = 0\hat{i} + 0\hat{j} + 12\hat{k}\ m$$

$$\underset{\sim}{M}_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 12 \\ \frac{8}{\sqrt{304}} & \frac{24}{\sqrt{304}} & \frac{-24}{\sqrt{304}} \end{vmatrix} kN \cdot m$$

$$= \left(\frac{-72\sqrt{19}}{19}\hat{i} + \frac{24\sqrt{19}}{19}\hat{j} + 0\cdot\hat{k}\right) kN \cdot m$$

Frames and Machines

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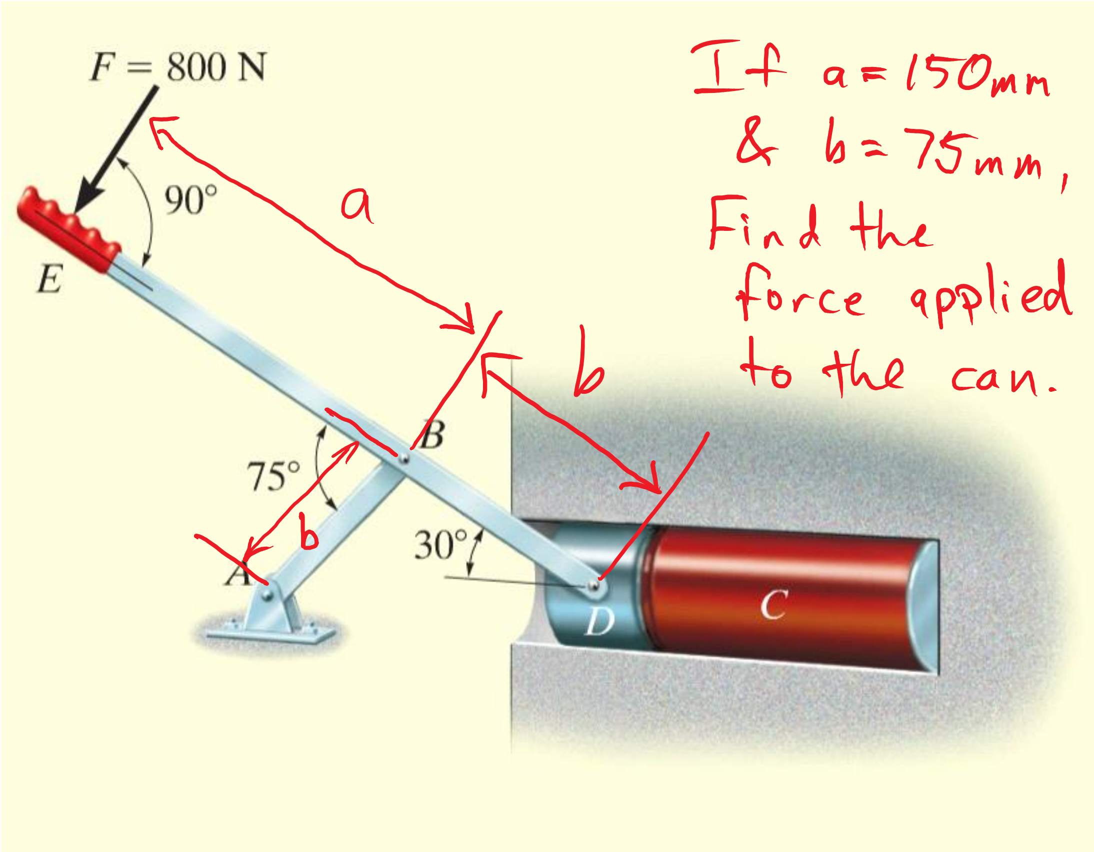
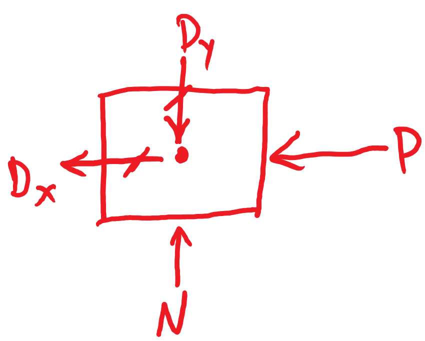


If $a = 150\,mm$ & $b = 75\,mm$, Find the force applied to the can.

FBD of piston at D



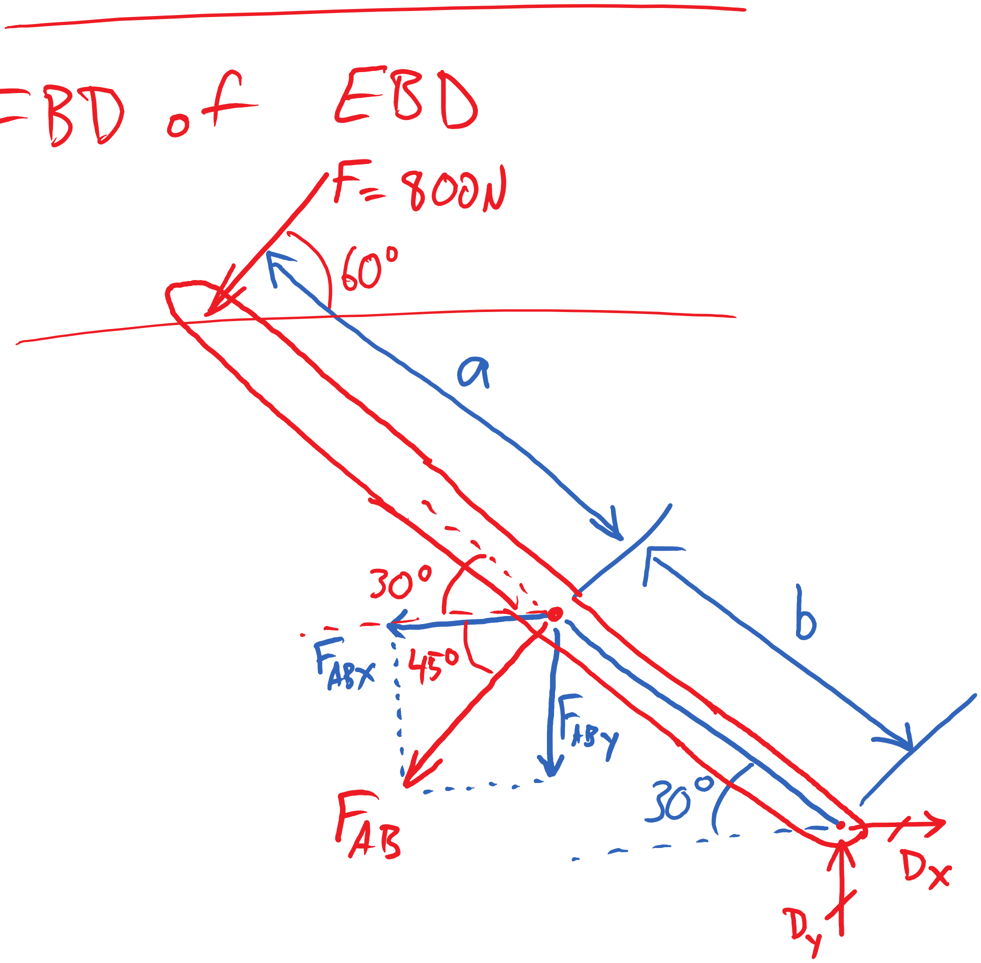
$\Sigma F_x = 0$

$\Rightarrow -D_x - P = 0$

$\Rightarrow P = -D_x$

See that link AB is a two-force member.

FBD of EBD



3 unknowns: F_{AB}, D_x, D_y

$$(\Sigma M)_D = 0$$

$$\Rightarrow b \cdot \cos(30^\circ) \cdot F_{ABy} + b \cdot \sin(30^\circ) \cdot F_{ABx} + F \cdot (a+b) = 0$$

$$F_{ABx} = F_{AB} \cdot \cos(45^\circ) = \frac{\sqrt{2}}{2} \cdot F_{AB}$$

$$F_{ABy} = F_{AB} \cdot \sin(45^\circ) = \frac{\sqrt{2}}{2} \cdot F_{AB}$$

$$\left(b\cdot\frac{\sqrt{3}}{2}\right)\cdot F_{AB}\cdot\frac{\sqrt{2}}{2}+\left(b\cdot\frac{1}{2}\right)\cdot F_{AB}\cdot\frac{\sqrt{2}}{2}+F\cdot(a+b)=0$$

$$b\cdot F_{AB}\cdot\left(\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}\right)=-F\cdot(a+b)$$

$$F_{AB}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)=-F\cdot\left(\frac{a+b}{b}\right)$$

$$F_{AB}=-F\cdot\frac{4(a+b)}{b(\sqrt{2}+\sqrt{6})}$$

$$=-(800N)\frac{4(225mm)}{(75mm)(\sqrt{2}+\sqrt{6})}$$

$$=-2485N$$

$$\Sigma F_x=0 \Rightarrow -F\cdot\cos(60^\circ)-F_{AB}\frac{\sqrt{2}}{2}+D_x=0$$

$$\Rightarrow D_x=F\cdot\frac{1}{2}+F_{AB}\cdot\frac{\sqrt{2}}{2}$$

$$D_x=(800N)\cdot\frac{1}{2}+(-2485N)\cdot\frac{\sqrt{2}}{2}$$

$$=-1357N$$

$$\Rightarrow \boxed{P=-D_x=1357N}$$